Rättningsmall

# General relativity: Bonustest II

May 9, 2018

Give your answers on this sheet. Unless stated in the question, you don't have to provide any reasoning or justifications for your answers. You can answer in english or in swedish.

Max: 14 p. At least 7 p gives 1 point to the exam. At least 11 p gives 2 points to the exam.

1. $A_{\alpha\beta}{}^{\gamma}$ is a rank 3 tensor. Insert indices below to obtain the correct transformation law to new coordinates $\alpha'$, $\beta'$, $\gamma'$. (1 p)

$$A_{\alpha'\beta'}{}^{\gamma'} = \frac{\partial x^{\alpha}}{\partial x^{\alpha'}}\frac{\partial x^{\beta}}{\partial x^{\beta'}}\frac{\partial x^{\gamma'}}{\partial x^{\gamma}} A_{\alpha\beta}{}^{\gamma}$$

2. The covariant derivative of a vector $\boldsymbol{a}$, expressed both with index upstairs and index downstairs, is

$$\nabla_\alpha a^\beta = \partial_\alpha a^\beta + \Gamma^\beta{}_{\alpha\gamma} a^\gamma$$

$$\nabla_\alpha a_\beta = \partial_\alpha a_\beta - \Gamma^\gamma{}_{\alpha\beta} a_\gamma$$

What is the corresponding expression for the covariant derivative of the rank 3 tensor $A_{\alpha\beta}{}^{\gamma}$? (2 p)

$$\nabla_\alpha A_{\beta\gamma}{}^{\delta} = \partial_\alpha A_{\beta\gamma}{}^{\delta} - \Gamma^{\mu}{}_{\alpha\beta} A_{\mu\gamma}{}^{\delta} - \Gamma^{\mu}{}_{\alpha\gamma} A_{\beta\mu}{}^{\delta} + \\ + \Gamma^{\delta}{}_{\alpha\mu} A_{\beta\gamma}{}^{\mu}$$

3. What kind of object is each of the following quantities? Mark the correct alternative and fill in the appropriate rank! (All correct – 4 p; 6 correct – 3 p; 5 or 4 correct – 2 p. 3 or 2 correct – 1 p.)

(a) total energy of a particle

☐ tensor of rank __ ☒ comp. of tensor of rank 1 ☐ neither

(b) energy density

☐ tensor of rank __ ☒ comp. of tensor of rank 2 ☐ neither

(c) the Einstein tensor

☒ tensor of rank 2 ☐ comp. of tensor of rank __ ☐ neither

(d) partial derivative of a vector field: $\partial_\alpha a^\beta$

☐ tensor of rank __ ☐ comp. of tensor of rank __ ☒ neither

(e) partial derivative of a scalar field: $\partial_\alpha \Phi$

☒ tensor of rank 1 ☐ comp. of tensor of rank __ ☐ neither

(f) covariant derivative of a vector field: $\nabla_\alpha a^\beta$

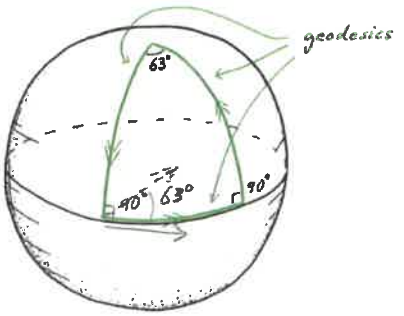
☒ tensor of rank 2 ☐ comp. of tensor of rank __ ☐ neither

(g) Christoffel symbol $\Gamma^\alpha{}_{\beta\gamma}$

☐ tensor of rank __ ☐ comp. of tensor of rank __ ☒ neither

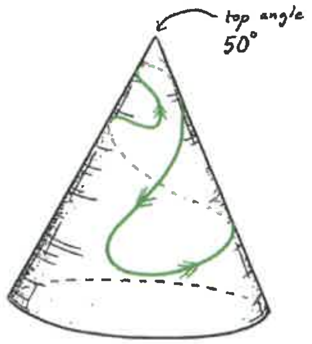
4. What are the results of parallel transporting a vector around the indicated loops on the following surfaces? Answer with the rotation angle (if any) as well as with the sense of rotation (clockwise or anti-clockwise). (3 p)

(a)




$63°$ counter clockwise

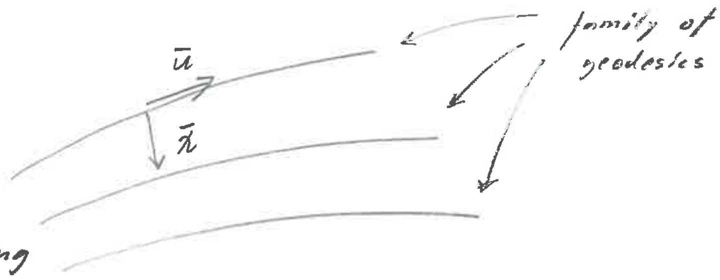
(b)




0° (surface is flat except at the tip, and the loop does not enclose that point)

5. The following two expressions ((a) and (b)) involving the Riemann tensor show its geometrical significance in two different ways. Explain the geometrical meaning of each of the expressions in a few words. Also, explain the meaning of each object involved. (A simple drawing in each case is recommended.) (4 p)

(a) $\nabla_u \nabla_u \chi^\alpha \;=\; -R^\alpha{}_{\beta\gamma\delta} u^\beta \chi^\gamma u^\delta$




$\bar{u}$ – tangent vectors to geodesics

$\bar{\chi}$ – separation vector between neighbouring geodesics

$\nabla_{\bar{u}} \nabla_{\bar{u}} \bar{\chi}$ is the acceleration of the separation vector along the geodesics, telling how the geodesics are converging or diverging

(b) $\delta A^\mu \;=\; -R^\mu{}_{\nu\alpha\beta} \delta S^{\alpha\beta} A^\nu$

$\delta A^\mu$ – change in vector $A^\mu$ after it has been parallell transported around loop enclosing area element $\delta S^{\alpha\beta}$