

General relativity: Bonustest II

May 9, 2018

Give your answers on this sheet. Unless stated in the question, you don't have to provide any reasoning or justifications for your answers. You can answer in english or in swedish.

Max: 14 p. At least 7 p gives 1 point to the exam. At least 11 p gives 2 points to the exam.

1. $A_{\alpha\beta\gamma}$ is a rank 3 tensor. Insert indices below to obtain the correct transformation law to new coordinates α', β', γ' . (1 p)

$$A_{\alpha'\beta'\gamma'} = \frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'} A_{\alpha\beta\gamma}$$

2. The covariant derivative of a vector \mathbf{a} , expressed both with index upstairs and index downstairs, is

$$\nabla_{\alpha} a^{\beta} = \partial_{\alpha} a^{\beta} + \Gamma^{\beta}_{\alpha\gamma} a^{\gamma}$$

$$\nabla_{\alpha} a_{\beta} = \partial_{\alpha} a_{\beta} - \Gamma^{\gamma}_{\alpha\beta} a_{\gamma}$$

What is the corresponding expression for the covariant derivative of the rank 3 tensor $A_{\alpha\beta\gamma}$? (2 p)

$$\nabla_{\alpha} A_{\beta\gamma}{}^{\delta} =$$

3. What kind of object is each of the following quantities? Mark the correct alternative and fill in the appropriate rank! (All correct – 4 p; 6 correct – 3 p; 5 or 4 correct – 2 p. 3 or 2 correct – 1 p.)

(a) total energy of a particle

tensor of rank ___ comp. of tensor of rank ___ neither

(b) energy density

tensor of rank ___ comp. of tensor of rank ___ neither

(c) the Einstein tensor

tensor of rank ___ comp. of tensor of rank ___ neither

(d) partial derivative of a vector field: $\partial_{\alpha} a^{\beta}$

tensor of rank ___ comp. of tensor of rank ___ neither

(e) partial derivative of a scalar field: $\partial_{\alpha} \Phi$

tensor of rank ___ comp. of tensor of rank ___ neither

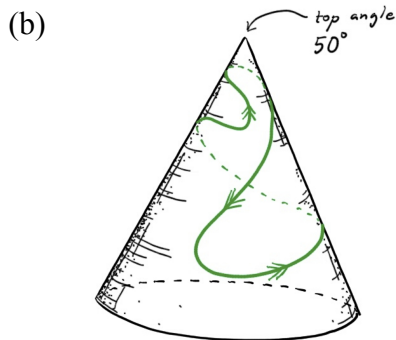
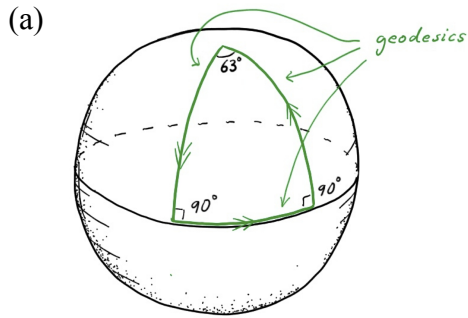
(f) covariant derivative of a vector field: $\nabla_{\alpha} a^{\beta}$

tensor of rank ___ comp. of tensor of rank ___ neither

(g) Christoffel symbol $\Gamma^{\alpha}_{\beta\gamma}$

tensor of rank ___ comp. of tensor of rank ___ neither

4. What are the results of parallel transporting a vector around the indicated loops on the following surfaces? Answer with the rotation angle (if any) as well as with the sense of rotation (clockwise or anti-clockwise). (3 p)



5. The following two expressions ((a) and (b)) involving the Riemann tensor show its geometrical significance in two different ways. Explain the geometrical meaning of each of the expressions in a few words. Also, explain the meaning of each object involved. (A simple drawing in each case is recommended.) (4 p)

(a)
$$\nabla_u \nabla_u \chi^\alpha = -R^\alpha_{\beta\gamma\delta} u^\beta \chi^\gamma u^\delta$$

(b)
$$\delta A^\mu = -R^\mu_{\nu\alpha\beta} \delta S^{\alpha\beta} A^\nu$$