General relativity: Bonustest I

April 15, 2019

Give your answers on this sheet. Unless stated in the question, you don't have to provide any reasoning or justifications for your answers. You can answer in english or in swedish.

Max: 16 p. At least 8 p gives 1 point to the exam. At least 12 p give 2 points to the exam.

1. Here are three line elements, all relevant in the vacuum outside a spherically symmetric star with mass M. One of them gives the exact spacetime geometry for this situation, and the other two are approximations. Which one is the exact one? For those that are approximations, state the conditions or limits under which they hold! (3 p)

   (I) $$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + \left(1+\frac{2M}{r}\right)dr^2 + r^2 d\Omega^2$$

   — holds in weak field limit, that is, $r \gg 2M$

   (II) $$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + \left(1-\frac{2M}{r}\right)^{-1}dr^2 + r^2 d\Omega^2$$

   — exact !

   (III) $$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + dr^2 + r^2 d\Omega^2$$

   — holds in weak field limit $r \gg 2M$ and non-relativistic trajectories $v \ll 1$

2. Consider a particle with 4-momentum
   $$p^\alpha = (2, -1, 1, 0)$$
   and an observer with 4-velocity
   $$u^\alpha = \left(\frac{5}{4}, \frac{3}{4}, 0, 0\right)$$
   (Both vectors are expressed in an orthonormal frame at the same point in spacetime.)

   (a) What is the speed (that is, the ordinary spatial 3-speed) of the observer in the given frame? (1 p)

   $|\vec{v}| = 0,6$

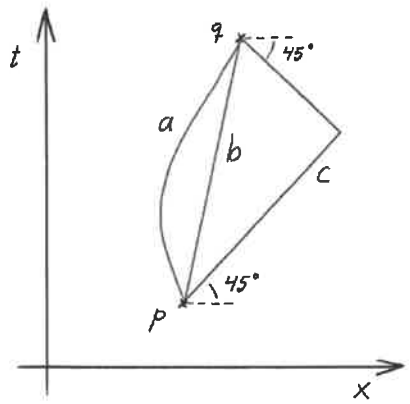
   (b) What is the mass of the particle? (1 p)

   $m = \sqrt{-p^\alpha p_\alpha} = \sqrt{2}$

   (c) What is the total energy of the particle as measured by the observer? (1 p)

   $E = -p^\alpha u_\alpha = \frac{13}{4}$

3. The spacetime diagram shows two events p and q in Minkowski space, and three different paths a, b and c connecting them. Order the paths *from the shortest to the longest*! Also, give a short explanation of your answer (a few words are enough). (2 p)



c , a , b

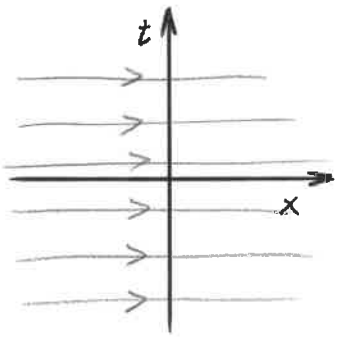
c is lightlike, and has length 0.

b is longest, since it is the straight path from p to q.

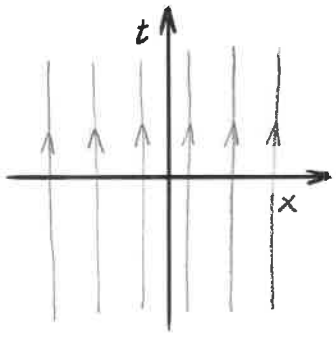
4. Consider 1+1 dimensional Minkowski space:

$$ds^2 = -dt^2 + dx^2$$

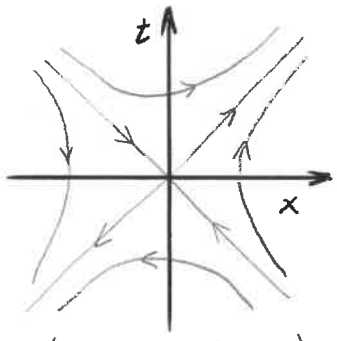
This spacetime has three independent Killing fields. Sketch the field lines of these (including arrows) in the diagrams below. (3 p)



(time translation)



(x-translation)



(Lorentz-boost)

5. Mark for each of the following statements if they are true or false, by making a ring around the correct alternative. (5 p) (Each statement gives +1 p for a correct answer; –1 p for an incorrect answer; 0 p for no answer. The problem as a whole cannot give negative points.)

(a) A clock at the top of Mount Everest is running slow compared to a clock in the valley below. — True (False)

(b) A clock near the floor of *a freely falling laboratory* is running slow compared to a clock near the ceiling of the same laboratory. — True (False)

(c) A metric is diagonal if and only if the coordinate lines are orthogonal. — (True) False

(d) A lightlike (null) curve in a 4 dimensional spacetime is necessarily also a geodesic. — True (False)

(e) A single Killing flow line can *not* be spacelike in one place and timelike in another. — (True) False