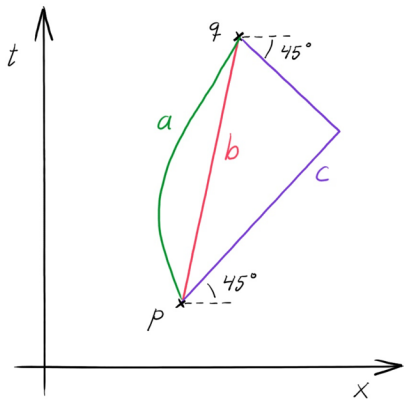
General relativity: Bonustest I

April 11, 2018

Give your answers on this sheet. Unless stated in the question, you don't have to provide any reasoning or justifications for your answers. You can answer in english or in swedish.

Max: 16 p. At least 8 p gives 1 point to the exam. At least 12 p give 2 points to the exam.

1. The spacetime diagram shows two events p and q in Minkowski space, and three different paths a, b and c connecting them. Order the paths *from the shortest to the longest*! Also, give a short explanation of your answer (a few words are enough). (2 p)



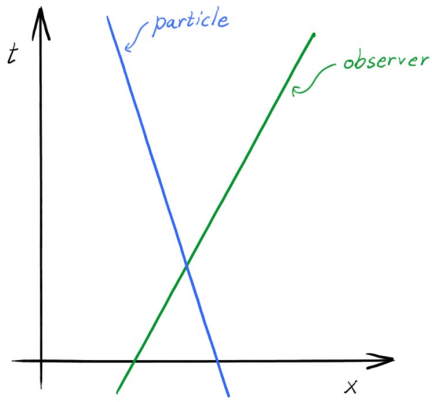
2. Consider a particle with 4-momentum

$$p^\alpha = (2, -1, 0, 0)$$

and an observer with 4-velocity

$$u^\alpha = a\,(5, 4, 0, 0)$$

where a is a normalization constant. (The metric is Minkowski, and both 4-vectors are given in ordinary Cartesian coordinates.)



(a) What is the value of a? (1 p)

(b) What is the speed of the observer? (1 p)

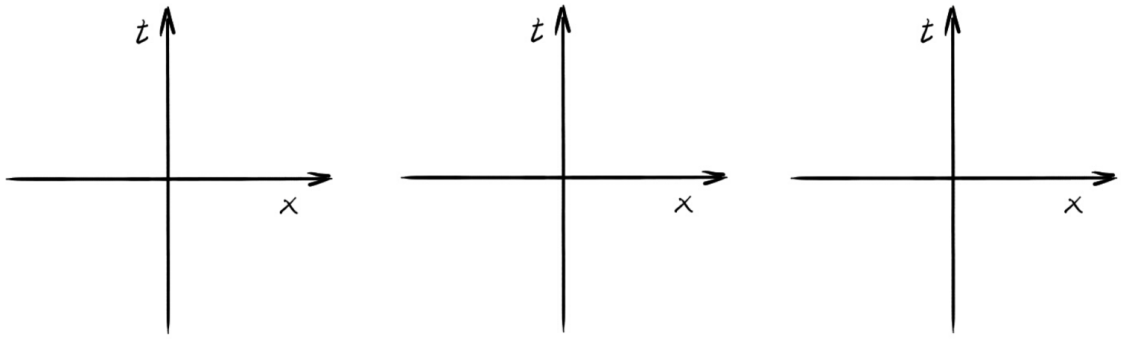
(c) What is the mass of the particle? (1 p)

(d) What is the total energy of the particle as measured by the observer? (1 p)

3. Consider 1+1 dimensional Minkowski space:

$$ds^2 = -dt^2 + dx^2$$

This spacetime has three independent Killing fields. Sketch the field lines of these (including arrows) in the diagrams below. (3 p)



4.

(a) What is, in general relativity, meant by an inertial frame? (1 p)

(b) What is the crucial difference between an inertial frame in special relativity and one in general relativity? (1 p)

5. Mark for each of the following statements if they are true or false, by making a ring around the correct alternative. (5 p) (Each statement gives +1 p for a correct answer; –1 p for an incorrect answer; 0 p for no answer. The problem as a whole cannot give negative points.)

(a) The basis vectors $(\mathbf{e}_r)^\alpha = (\ 1\ ,\ 0\ )$ and $(\mathbf{e}_\varphi)^\alpha = (\ 0\ ,\ 1\ )$ in polar coordinates r, φ with line element $ds^2 = dr^2 + r^2\, d\varphi^2$ constitute an orthonormal basis. True False

(b) The 4-velocity of a freely falling laboratory, together with three orthogonal spacelike vectors of length one attached to the laboratory, constitute an orthonormal basis. True False

(c) A clock at the top of Mount Everest runs fast compared to a clock in the valley below. True False

(d) A clock near the ceiling of a freely falling laboratory runs fast compared to a clock near the floor of the same laboratory. True False

(e) A metric is diagonal if and only if the coordinate lines are orthogonal. True False